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Faculty of Science, Department of Physics
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Lecture 9: Galilean Invariance
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CONTENT

Introduction; Frame of reference; Galilean invariance; Galilean transformation equations; Galilean velocity transformation; Galilean acceleration transformation; Universal gravitation.

INTRODUCTION

In the beginning of the year 1905, Albert Einstein was an unknown 25 year old clerk in the Swiss patent office. By the end of that year, he had published papers of great importance on Brownian motion, photoelectric effect and special theory of relativity, one of which he was awarded the Nobel Prize in Physics.

The special theory of relativity has made wide-ranging changes regarding our understanding of nature, but Einstein based it on just two simple postulates. The first postulate, principle of relativity, also called Galilean relativity or Galilean Invariance forms the basis of discussion in this chapter.

FRAME OF REFERENCE

The basic idea of motion is *change of position with time*. We say that an object is moving if it occupies different positions in space at different times. If an object does not change its position in space as time flows, we say that the object is at rest.

Our perception tells us that space exists as a stationary, permanent and absolute background in which objects can be placed or moved without any interaction; and that time is also absolute and flows on without relation to the presence of any physical object or event. According to Newtonian absolute space and time, motion of an object is characterized by a change of position in space as time evolves.

However, since this absolute space cannot be seen, how then do we fix the position of an object in this space? Newton regarded the distant stars in the sky at rest, and defined the position of other objects in relation to them. According to astronomers, none of the cosmological objects are at rest in this 'absolute space' with respect to which we can define the position of another object. Therefore, at any instant of time, the position of an object is always defined by the 'observer' with respect to himself. An **observer**, therefore, first defines a local space in which the positions of other objects are then described. This local space to which an observer is attached is called a *frame of reference*.

Hence, in order to describe physical events that occur in space and time such as the motion of bodies, we introduce a coordinate system. Its spatial and temporal coordinates can now specify a space-time event by placing an observer at the origin of the coordinate system. Thus, a **frame of reference** can be defined as an *arbitrary set of coordinates (space-time) that describes the position, velocity and acceleration of objects*.

Generally, we use the rectangular (Cartesian) coordinates (x, y, z) to determine the position of an object in space. Other coordinate systems are spherical polar coordinates (r, θ, ϕ) and cylindrical coordinates (ρ, ϕ, z) . The figure below shows the position of a particle P described by the coordinates x, y and z in a frame of reference.

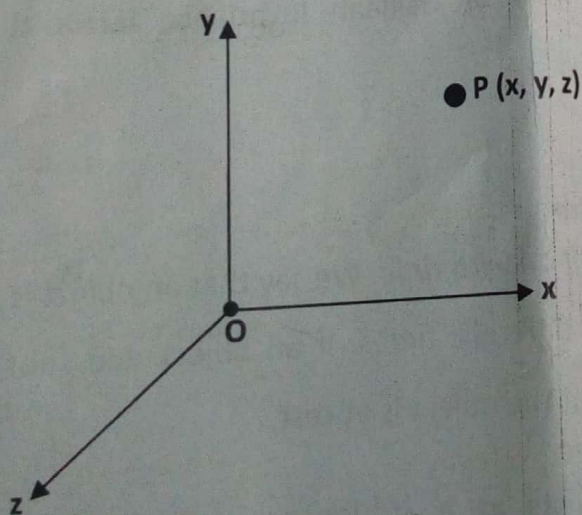


Figure 1: Frame of reference

GALILEAN INVARIANCE

Galilean invariance or **Galilean relativity** states that *the laws of motion are the same in all inertial frames of reference*. An **inertial frame of reference** is a three dimensional coordinate

system which travel at constant velocity. It is a reference frame in relative motion to absolute space. In such a frame, the object does not accelerate.

Galilee Galileo first described this principle using the example of a ship travelling at constant velocity, without rocking, on a smooth sea; any observer performing experiments below the deck would not be able to tell whether the ship was moving or stationary.

Among the axioms from Newton's theory are:

- There exists an absolute space in which Newton's laws are true.
- All inertial frames share a universal time.

Galilean Transformation Equations

Suppose we know from experiment that these laws of mechanics are true in one frame of reference, how do they look in another frame, moving with respect to the first frame? To find out, we figure out how to get from position, velocity and acceleration in one frame to these quantities in the second frame.

Note: The two frames must have a constant relative velocity, otherwise the law of inertia would not hold in both of them.

Suppose S' is proceeding relative to S at a speed v along the x -axis:

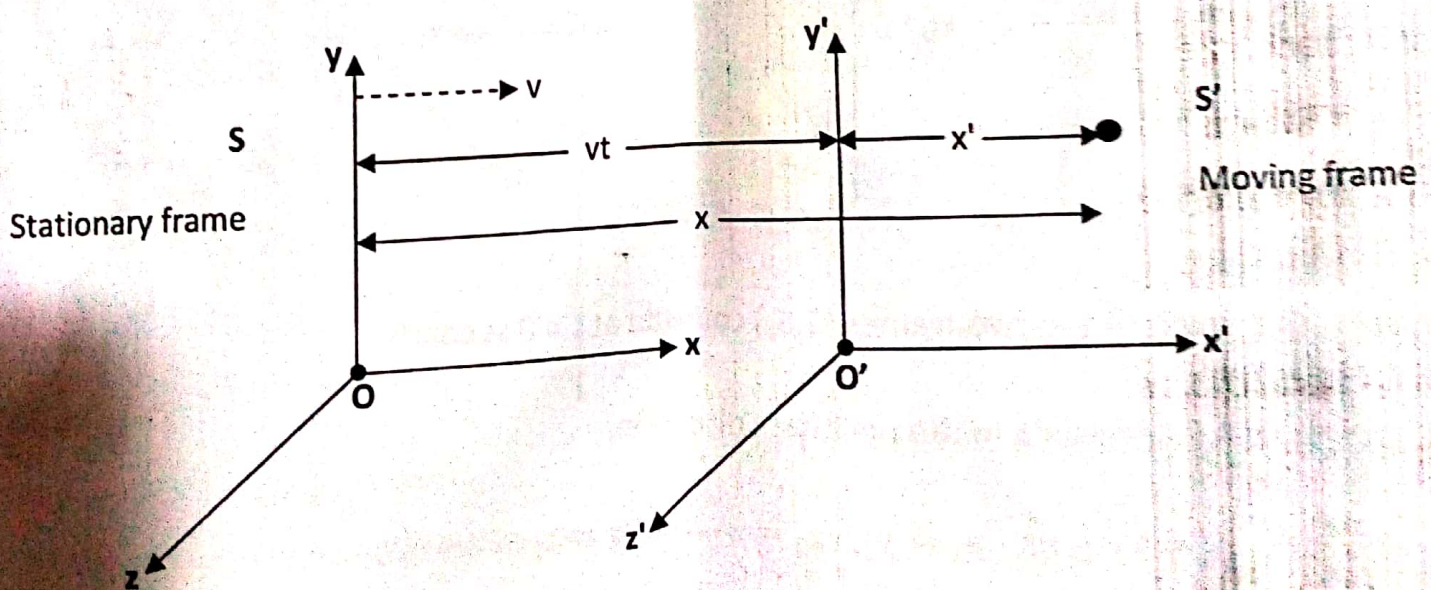


Figure 2: S and S' inertial frame of reference

We see from the diagram above that $x = x' + vt$. Also, $y = y'$ and $z = z'$ since there are no movements in these directions and $t = t'$ since time is universal. Thus, (x, y, z, t) in S frame corresponds to (x', y', z', t') in S' frame, where:

$$x' = x - vt$$

$$y' = y$$

$$z' = z$$

$$t' = t$$

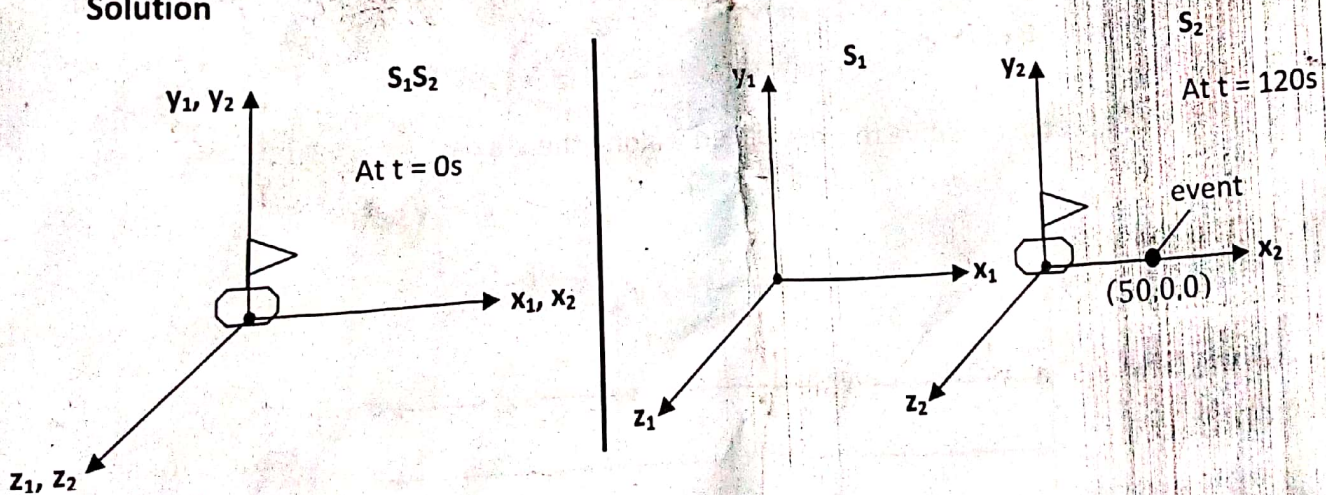
Galilean Transformation

Equations

That is how position transforms and is known as Galilean Transformations.

Example 1: A boat that is initially stationary begins to move with a velocity of 10m/s with respect to x - axis in the positive direction. Let the water on which it moves be reference frame 1 and the boat represents reference frame 2. Assume that reference frame 1 is always stationary and at $t = 0$ seconds, the two frames coincide. If we are in reference frame 2 and an event takes place by the boat at point $(x_2 = 50, y_2 = 0, z_2 = 0)$ after the boat travelled for 120s, find the equivalent position of this event in reference frame 1.

Solution



Initially, the origin of the two frames (S_1, S_2) coincide at $t = 0$ seconds

Since the boat is travelling in the positive x direction,

$$x_1 = x_2 + vt, \quad y_1 = y_2, \quad z_1 = z_2 \quad \text{and} \quad t_1 = t_2$$

However, at $t = 120s$, $x_2 = 50, y_2 = 0, z_2 = 0$

Therefore, $x_1 = x_2 + vt$

$$x_1 = 50 + 10(120) = 50 + 1200$$

$$x_1 = 1250\text{m}$$

Thus, point (50, 0, 0) in frame 2 \equiv point (1250, 0, 0) in frame 1

Galilean Velocity Transformation

The velocity u'_x in S' frame in the x' direction as measured by an observer is given as:

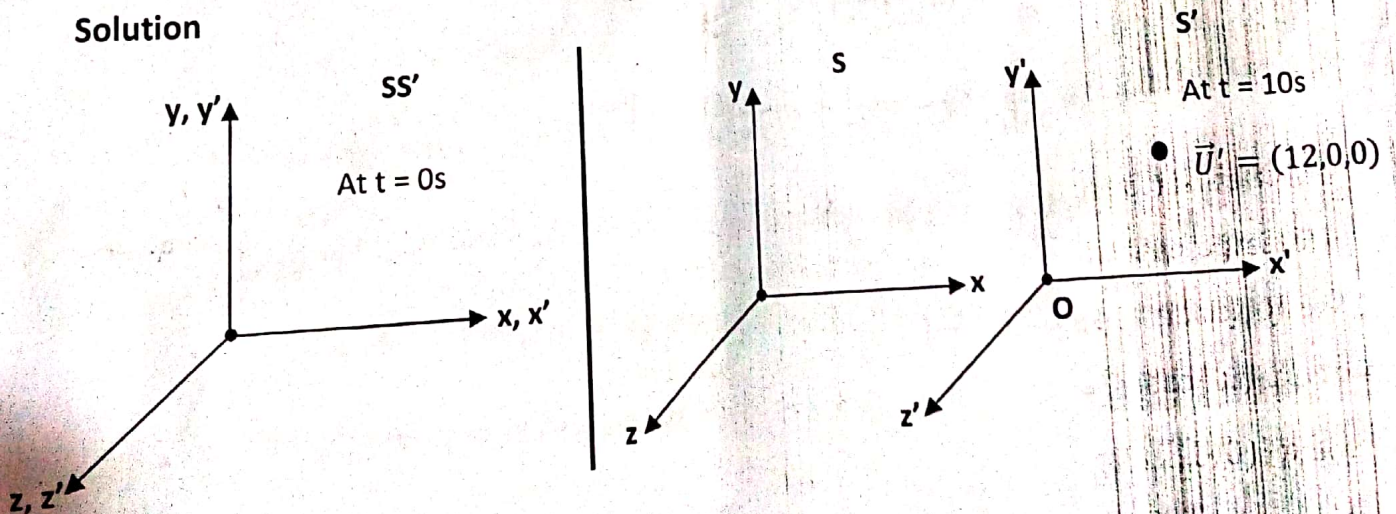
$$u'_x = \frac{dx'}{dt'} = \frac{dx}{dt} \quad (\text{since } t' = t)$$

$$u'_x = \frac{d}{dt}(x - vt) = \frac{dx}{dt} - v$$

Therefore, $u'_x = u_x - v$ (Galilean velocity transformation)

Example 2: At $t = 0$ seconds, two reference frames consisting of xyz plane coincide such that the origin of one is found at the origin of the second. However, as time progresses, the second reference frame moves in the positive direction along x -axis with velocity of 37m/s , while the first frame of reference remains stationary. At a time of $t = 10$ seconds, a particle within the second reference frame has a velocity given by vector $\vec{U}' = (12, 0, 0) \text{ m/s}$. Find the equivalent velocity U' of the particle with respect to the first reference frame.

Solution



Galilean velocity transformation equations are:

$$\vec{U}_x = \vec{U}'_x + v, \quad \vec{U}_y = \vec{U}'_y, \quad \vec{U}_z = \vec{U}'_z$$

$$\vec{U} = (\vec{U}_x, \vec{U}_y, \vec{U}_z) = (\vec{U}'_x + v, \vec{U}'_y, \vec{U}'_z) = (12 + 37, 0, 0)$$

$\therefore \vec{U} = (49, 0, 0) \text{ m/s}$ (Velocity of particle w.r.t. reference frame 1)

Galilean Acceleration Transformation

How does acceleration transform?

$$a'_x = \frac{du'_x}{dt'} = \frac{du'_x}{dt}$$

$$a'_x = \frac{d}{dt}(u_x - v) = \frac{du_x}{dt} - \frac{dv}{dt}$$

Since v is constant, $\frac{dv}{dt} = 0$

Therefore, $a'_x = a_x$

This means that the acceleration is the same in both frames. Thus, we say that acceleration in both frames is Galilean Invariant. Velocity is not invariant because it differs in both frames by a constant factor i.e. the relative velocity of both frames.

We can also show that the law of conservation of momentum (Newton's second law of motion) $F = ma$, is invariant under Galilean transformation.

In the S' frame of reference, $F' = ma'_x$

$$F' = m \frac{du'_x}{dt} = m \frac{d}{dt} \left| \frac{dx'}{dt} \right|$$

$$F' = m \frac{d}{dt} \left| \frac{d}{dt}(x - vt) \right| = m \frac{d}{dt} \left| \frac{dx}{dt} - v \right|$$

$$F' = m \frac{d}{dt} [u_x - v]$$

$$F' = m \left| \frac{du_x}{dt} - \frac{dv}{dt} \right|$$

But $\frac{dv}{dt} = 0$

Therefore, $F' = ma_x = F$

Universal Gravitation

Newton's law of universal gravitation states that a particle attracts every other particle in the universe with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centres. This gravitational force is the same as encountered in Newton's second law of motion, where:

$$F = ma = \frac{GMm}{r^2}$$

Where G is gravitational constant and r is the distance of separation between the masses

Since force, F is Galilean invariant, we can conclude that the law of universal gravitation is also Galilean invariant, supporting the notion that all physical laws are invariant under Galilean transformation.

Example 3: Show that the law of conservation of linear momentum for collision is Galilean invariant, given that $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

Solution

From Galilean velocity transformation equation,

$$u_1 = u'_1 + v$$

$$u_2 = u'_2 + v$$

$$v_1 = v'_1 + v$$

$$v_2 = v'_2 + v$$

Substituting these equations into the equation for momentum conservation, we have

$$m_1(u'_1 + v) + m_2(u'_2 + v) = m_1(v'_1 + v) + m_2(v'_2 + v)$$

$$m_1u'_1 + m_1v + m_2u'_2 + m_2v = m_1v'_1 + m_1v + m_2v'_2 + m_2v$$

Collecting like terms and subtracting, we have

$$m_1u'_1 + m_2u'_2 = m_1v'_1 + m_2v'_2$$

This shows that the law of conservation of linear momentum is Galilean Invariant i.e. remain the same in different inertial frames of reference.

↑

Exercise

7.1 A girl is riding a bicycle along a straight road at constant speed, and passes a friend standing at a bus stop (event 1). At a time of 60s later, the friend catches a bus (event 2). If the distance separating the events is 126m in the frame of the girl on the bicycle, what is the bicycle's speed?
(Ans: 2.10m/s)

7.2 A police car travelling on a straight road at a constant speed of 30m/s catches up with a truck travelling at a constant speed of 20m/s. The police car reaches the truck 25s after it is first sighted. How far ahead of the police car was the truck when it was first sighted?
(Ans: 250m)

7.3 Show that the law of conservation of kinetic energy is Galilean Invariant.

$$\text{Hint: } \left(\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \right)$$